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Exploring Conceptual Understanding of Trigonometric Function Transformation through GeoGebra-Assisted Project-Based Learning: A Pilot Case Study

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Abstract. Trigonometric function transformation, governed by the parameters in $f(x) = A \sin(Bx + C) + D$, is widely reported as a persistent source of student misconceptions because interpreting it requires the simultaneous coordination of several abstract parameters. This pilot case study describes the conceptual understanding demonstrated by one mathematics education student regarding the parameters of amplitude, period, phase shift, and vertical shift after completing a GeoGebra-assisted exploration task structured around a four-stage Project-Based Learning (PjBL) model. A qualitative document analysis was conducted on the participant's worksheets, which contained observation tables, GeoGebra screenshots, written analyses, and reflections for each parameter, following an approach that isolated a single parameter before requiring the combination of all parameters. The participant's responses were compared against commonly reported misconception indicators for trigonometric transformation drawn from the literature, including confusion between the sign and magnitude of the amplitude, treating the period-frequency relationship as directly rather than inversely proportional, errors in the direction of the phase shift, and failure to divide the phase constant by the frequency coefficient when combining parameters. The analysis showed that the participant correctly identified all four parameter-graph behavior relationships, including the phase-by-frequency division step that is most frequently reported as a source of student error, and no clear misconception was found in this case. These findings tentatively suggest that staging GeoGebra exploration from single parameters toward combined parameters within a PjBL structure may support the construction of accurate concepts, although the limited scope of a single participant constrains the generalizability of the findings. This study was primarily intended to pilot the worksheet design and analytical framework in preparation for a broader study involving a larger number of participants.

Keywords: *conceptual understanding; GeoGebra; misconception; project-based learning; trigonometric function transformation*

INTRODUCTION

Trigonometric functions occupy a fundamental position in secondary and higher education mathematics curricula and form the basis for further study in physics, engineering, and other quantitative fields. Among the topics within this domain, the transformation of the general sinusoidal function $f(x) = A \sin(Bx + C) + D$ is considered conceptually challenging because students must coordinate four parameters simultaneously: amplitude (A), frequency (B), phase shift (C), and vertical shift (D). Prior studies have noted that trigonometry has long been regarded as one of the more difficult school mathematics topics, and that learning difficulties and misconceptions in this domain have been widely documented across various educational contexts [1].

A recurring finding in the literature is that trigonometry instruction relying primarily on rote memorization of formulas, without accompanying visual or dynamic representations, tends to leave the relationships between parameters and graphical behavior poorly understood. Building an accurate mental connection between algebraic expressions and their graphical consequences—for example, linking a change in the coefficient B to a change in the period—has been reported as a major source of difficulty for many students [2]. Dynamic mathematics software such as GeoGebra has been proposed as one way to bridge this gap, as it allows learners to manipulate parameters through sliders and observe the resulting graphical changes directly, rather than inferring the relationship solely from a static formula [1], [2].

Project-Based Learning (PjBL) offers a complementary instructional structure for this type of exploration. Rather than passively receiving rules, students actively construct understanding through a sequence of guided tasks and reflection, consistent with constructivist views of learning [3]. In the context of Indonesian higher education, PjBL has been reported to be perceived positively by mathematics education students as a means of strengthening conceptual understanding as well as creativity and collaborative problem-solving skills [4]. Combining dynamic GeoGebra-based exploration with a staged

PjBL worksheet—moving from single-parameter exploration toward the integration of all parameters—is a sensible design for supporting robust conceptual understanding; however, documented evidence of how an individual learner's reasoning actually develops across such a staged sequence of tasks at the higher education level remains limited.

Most empirical research on GeoGebra and trigonometric misconceptions relies on classroom-scale quantitative pre-test/post-test designs at the secondary school level [1], [2]. More detailed document-based analyses of how a single learner's written work, tables, and reflections reveal—or fail to reveal—particular misconception patterns during a staged exploration task remain relatively rare, particularly in the context of pre-service mathematics teacher education. This gap is relevant because pre-service teachers' conceptual understanding of trigonometric transformation has direct implications for how they will later teach this topic.

This study therefore aims to (1) describe, as a pilot case study, the conceptual understanding demonstrated by one mathematics education student throughout a four-stage GeoGebra-assisted PjBL exploration of amplitude, period, phase shift, and their combination, and (2) pilot an analytical framework, derived from literature-based misconception indicators, to identify potential misconceptions in this type of task. The findings are intended to provide a basis for developing instruments and procedures for a broader follow-up study.

METHODS

Research Context

This pilot case study was conducted in the Trigonometry course (MTC203), Department of Mathematics Education, Faculty of Teacher Training and Education, Universitas Cipasung, Tasikmalaya, Indonesia, during the 2026 academic year. As part of the midterm assessment, students completed a Project-Based Learning (PjBL) task titled “Exploring Graphs of Trigonometric Functions Using GeoGebra,” designed around the general function $f(x) = A \sin(Bx + C) + D$. The analysis in this article focuses on the worksheet completed by one participating student (hereafter referred to as Participant 1, P1), used with the student's permission for academic reporting purposes.

Procedure

Task design (Tasks A–D)

The tasks were structured in four sequential stages using GeoGebra Classic. Task A isolated the amplitude parameter A , holding $B = 1$, $C = 0$, and $D = 0$ constant, and asked participants to record the maximum and minimum values for $A = 0.5, 1, 2, 3$, and -2 . Task B isolated the frequency parameter B ($A = 1$, $C = 0$, $D = 0$), requiring calculation of the theoretical period $T = 2\pi/B$ and a comparison between the radian and degree display modes for $B = 0.5, 1, 2, 4$. Task C isolated the phase shift parameter C ($A = 1$, $B = 1$, $D = 0$), requiring identification of the direction and magnitude of the shift for $C = -\pi, -\pi/2, 0, \pi/2, \pi$, as well as a comparison between $y = \sin(x + \pi/2)$ and $y = \cos(x)$. Task D required combining all four parameters to reproduce three target graph specifications defined in terms of amplitude, period, phase shift, and vertical shift. For each task, participants recorded an observation table, captured GeoGebra screenshots for each parameter value, wrote a brief analysis, and composed a reflection.

Data collection

The completed worksheets, comprising observation tables, GeoGebra screenshots, written analyses, and reflections for all four tasks, were collected as the primary data source for this pilot study.

Data Analysis

The worksheets were analyzed using qualitative document analysis. The participant's observations and written analyses for each task were compared against the mathematically correct parameter-graph behavior relationships. In parallel, the participant's responses were screened using a set of literature-based misconception indicators commonly reported in the learning of trigonometric transformation: (a) confusion between the effect of the sign of A and its magnitude, i.e., failure to distinguish amplitude from reflection; (b) treating the period and frequency (B) as directly rather than inversely proportional; (c) confusion regarding the direction associated with a positive or negative phase constant C ; and (d) failure to divide the phase constant C by the frequency coefficient B when determining the actual horizontal shift in the parameter combination task. Each task response was coded as consistent or inconsistent with the correct relationship and, if inconsistent, matched against the indicators listed above.

RESULTS AND DISCUSSION

Task A: Amplitude Exploration

P1's observation table (Table 1) shows accurate identification of the maximum and minimum values across all tested values of A, including the negative case ($A = -2$).

Table 1. Participant's observations of the effect of parameter A on the graph's maximum and minimum values.

Value of A	Max.	Min.	Period changes?	x-intercept changes?
0.5	0.5	-0.5	No	No
1	1	-1	No	No
2	2	-2	No	No
3	3	-3	No	No
-2	2	-2	No	No

Tugas A : Eksplorasi Amplitudo (Parameter A)

1. Metode Eksplorasi

Berisi Langkah-langkah menggunakan Geogebra (sesuai dengan petunjuk di Tugas A)

2. Hasil Eksplorasi Tugas A – Pengaruh Parameter A Terhadap Grafik

2.1 Tabel Hasil Pengamatan

Hasil pengamatan dari setiap nilai A dicatat dalam tabel berikut:

Setup awal : $f(x) = A \cdot \sin(x)$, set $B=1, C=0, D=0$

Nilai A	Nilai Maks.	Nilai Min.	Periode Berubah?	Perpotongan sb-x Berubah?
0,5	0,5	-0,5	Tidak	Tidak
1	1	-1	Tidak	Tidak
2	2	-2	Tidak	Tidak
3	3	-3	Tidak	Tidak
-2	2	-2	Tidak	Tidak

- Analisis A Negatif: Grafik akan terbalik (refleksi) terhadap sumbu-x. Jika $A=2$ grafik mulai naik, maka $A=-2$ grafik mulai turun.
- Kesimpulan: Nilai $|A|$ menentukan amplitudo (tinggi gelombang), sedangkan tanda A menentukan arah hadap/orientasi grafik (refleksi terhadap sumbu -x).

2.2 Screenshot Grafik

Berikut adalah hasil screenshot grafik untuk setiap nilai A yang diujikan:

- 1) Set nilai awal; $B=1, C=0, D=0$. Geser slider A dari -3 hingga 3.
- 2) Screenshot grafik untuk $A=0.5, 1, 2, 3$ secara berurutan.

Figure 1. GeoGebra screenshots for $A = 0.5, 1, 2$, and 3 ($B = 1, C = 0, D = 0$).

In the written analysis, P1 explicitly separated the role of the absolute value $|A|$ in determining the amplitude from the role of the sign of A in determining the graph's orientation, correctly stating that a negative A reflects the graph across the x-axis without changing either the amplitude or the period. This distinction corresponds to indicator (a) in the analytical framework; no confusion between amplitude magnitude and sign-based reflection was found.

Task B: Period Exploration

Table 2. Participant's observations of the effect of parameter B on the period, in radian and degree modes.

Value of B	$T = 2\pi/B$ (theoretical)	Measured T	$T = 360^\circ/B$ (degrees)	Graph denser?
0.5	4π	12.57	720°	No (stretched)
1	2π	6.28	360°	Normal
2	π	3.14	180°	Yes
4	0.5π	1.57	90°	Yes (very dense)

TUGAS B : EKSPLORASI PERIODE (Parameter B)**1. Metode Eksplorasi**

Berisi langkah-langkah menggunakan Geogebra (sesuai dengan petunjuk di Tugas B)

2. Hasil Eksplorasi Tugas B – Satuan Radian dan Derajat**2.1 Tabel Hasil Pengamatan**

Hasil pengamatan dari setiap nilai B dicatat dalam tabel berikut:

Setup awal : $f(x) = \sin(B \cdot x)$, set $A=1, C=0, D=0$

Nilai B	$T=2\pi/B$ (teoritis)	T terukur dari grafik	$T=360^\circ/B$ (derajat)	Grafik lebih rapat?
0,5	4π	12,57	720°	Tidak (meregang)
1	2π	6,28	360°	Normal
2	π	3,14	180°	Ya
4	$0,5\pi$	1,57	90°	Ya (sangat rapat)

- Kesimpulan: Semakin besar nilai B, periode T semakin kecil, artinya grafik semakin rapat/berulang lebih sering.

2.2 Screenshot Grafik

Berikut adalah hasil screenshot grafik untuk setiap nilai A yang diujikan:

- Set $A=1, C=0, D=0$. Geser slider B dari 0,5 hingga 4.
- Hitung periode teoritis: $T=2\pi/B$ dan eksplorasi dalam mode derajat: $T=360^\circ/B$.

Figure 2. GeoGebra screenshots comparing the period across values of B in degree mode.

P1 accurately calculated the theoretical period $T = 2\pi/B$ for each value of B and matched it with the graph density observed visually, concluding that the period and B are inversely proportional. This relationship is the one most frequently reported as prone to error (indicator b), yet P1's analysis and reflection both correctly formulated the inverse relationship, and even noted that changing the angle display mode from radians to degrees affects only the axis scale, not the underlying period relationship.

Task C: Phase Shift Exploration

Table 3. Participant's observations of the effect of parameter C on the direction and magnitude of the horizontal shift.

Value of C	Shift direction	Shift magnitude	Amplitude changes?	Period changes?
$-\pi$	Right	π	No	No
$-\pi/2$	Right	$\pi/2$	No	No
0	Unchanged	0	No	No
$\pi/2$	Left	$\pi/2$	No	No
π	Left	π	No	No

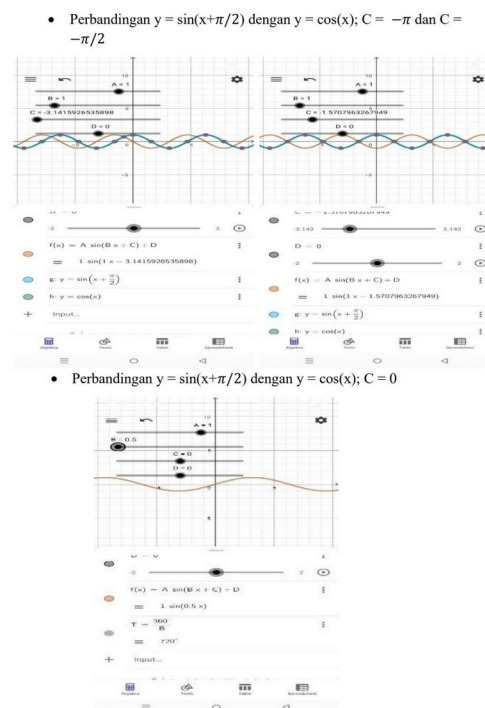


Figure 3. GeoGebra screenshots comparing $y = \sin(x + C)$ with $y = \cos(x)$ for various values of C .

P1 correctly identified that a positive C shifts the graph to the left and a negative C shifts it to the right (indicator c), and used GeoGebra to verify the identity $\sin(x + \pi/2) = \cos(x)$ through a direct graph overlay. In an extension activity, P1 also correctly determined that $C = 3\pi/2$ (or equivalently $-\pi/2$) yields $\sin(x + C) = -\cos(x)$, indicating that understanding of this phase relationship had been generalized beyond the specific worked example.

Task D: Combined Parameter Exploration

Table 4. Parameter values determined by the participant to satisfy three target graph specifications.

Graph	A	B	C	D
Graph 1 (amplitude 2, period π , phase $\pi/4$ left)	2	2	$\pi/2$	0
Graph 2 (amplitude 0.5, period 4π , vertical +1)	0.5	0.5	0	1
Graph 3 (free-form: sound wave pattern)	3	20	0	0

3. Analisis

3.1 Pengaruh Kombinasi Parameter

Berdasarkan hasil pengerjaan Tugas D, dapat dianalisis bahwa setiap parameter bekerja secara independen namun saling melengkapi dalam membentuk profil gelombang yang kompleks. Parameter A dan D mengontrol dimensi vertikal (tinggi dan posisi), sedangkan B dan C mengontrol dimensi horizontal (kerapatan dan titik awal). Ketepatan dalam menghitung nilai C sangat bergantung pada nilai B yang digunakan, karena pergeseran fase sebenarnya adalah hasil bagi antara C dan B.

4. Kesimpulan

Dari eksplorasi Tugas D, saya menyimpulkan bahwa:

- Untuk membangun grafik dengan spesifikasi tertentu, kita harus memahami hubungan matematis antar parameter, terutama hubungan B dengan periode dan C dengan pergeseran fase.
- GeoGebra mempermudah proses verifikasi nilai yang telah dihitung secara teoritis melalui perubahan visual yang instan.
- Parameter D adalah satu-satunya parameter yang dapat mengubah posisi seluruh grafik tanpa mengubah bentuk atau karakteristik gelombangnya.

5. Refleksi

Tugas D merupakan bagian yang paling menantang karena menuntut pemahaman integratif dari Tugas A, B, dan C. Saya menyadari bahwa kesalahan kecil pada satu parameter, seperti lupa membagi nilai pergeseran fase dengan nilai B, akan membuat seluruh grafik menjadi tidak akurat. GeoGebra sangat membantu sebagai alat kendali mutu untuk memastikan hasil perhitungan saya sudah sesuai dengan target spesifikasi. Tantangan terbesar adalah memodelkan "Grafik 3" yang menyerupai gelombang suara, di mana saya harus bereksperimen dengan frekuensi (B) yang sangat tinggi agar mendapatkan visualisasi yang diinginkan.

Figure 4. GeoGebra screenshots for the three combined-parameter target graphs.

The parameter combination task is the point most frequently reported to reveal indicator (d), namely failure to divide the phase constant by the frequency coefficient. For Graph 1, P1 needed to produce a $\pi/4$ shift to the left with $B = 2$. P1's calculation explicitly determined C by solving $C/B = \pi/4$, yielding $C = \pi/2$, rather than directly setting C equal to $\pi/4$. This division step was applied correctly, and P1's reflection explicitly identified it as the step most likely to produce an inaccurate graph if overlooked, indicating awareness of this potential source of error rather than an unrecognized misconception.

Discussion

Across all four tasks, P1's responses were consistent with the mathematically correct parameter-graph behavior relationships, and none of the four literature-based misconception indicators was observed in this case. Two aspects of the task design may account for this result. First, isolating one parameter at a time (Tasks A–C) before requiring their integration (Task D) allowed the participant to build an understanding of each parameter's effect before facing the additional cognitive demand of combination, consistent with the general rationale for using GeoGebra to link algebraic and graphical representations in stages [1], [2]. Second, the requirement to write an explicit textual analysis and reflection after each task likely encouraged the participant to articulate and re-examine relationships—such as the C/B division in Task D—that might otherwise be applied procedurally without verification.

This observation is broadly consistent with reports that PjBL supports actively and independently constructed understanding in mathematics education [3], [4], as well as with findings that GeoGebra-assisted learning can reduce certain trigonometric misconceptions, such as those related to periodicity, compared with formula-based instruction alone [1]. However, because this pilot study examined only one participant's work, it cannot be determined whether this result reflects the instructional design, the participant's individual prior readiness, or both. The absence of misconceptions in this case also means the study has not yet demonstrated the analytical framework's sensitivity on a sample with genuinely varied performance; this still needs to be tested once data from additional participants, including those potentially exhibiting errors, become available.

CONCLUSION

This pilot case study examined the conceptual understanding of one mathematics education student regarding trigonometric function transformation after completing a staged, GeoGebra-assisted PjBL exploration task. The participant correctly identified the roles of amplitude, period, phase shift, and their combination, including the phase-by-frequency division step that is often a source of student error, and no misconceptions from the literature-based framework were found in this case. These results tentatively support the feasibility of staging GeoGebra exploration from single to combined parameters within a PjBL structure; however, given its limited scope of a single participant, these findings should be read as illustrative rather than generalizable. This pilot study has fulfilled its purpose of piloting the worksheet design and analytical framework; a follow-up study involving a full class cohort together with pre/post diagnostic instruments is planned to map the prevalence of misconceptions more systematically and to evaluate the effectiveness of this instructional design.

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